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ANALOG COMPUTATION OF SOLUTIONS OF THE BLOCK EQUATIONS IN A WEAK FIELD. APPLICATION TO THE NUCLEAR SPIN GYRO. 2#

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ANALOG COMPUTATION OF SOLUTIONS OF THE BLOCH EQUATIONS IN A WEAK FIELD. APPLICATION TO THE NUCLEAR SPIN GYRO.

Guy Hanuise and Eric Varoquaux

SUMMARY

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Spin properties are briefly surveyed and the nuclear spin gyro principle is recalled. The Bloch equations relating to this device are then transformed with a view to solving them by means of an analog computer. The results are discussed, and some conditions for the design of nuclear spin gyros are derived. It is shown that using very low temperatures greatly increases the sensitivity. Author /41

INTRODUCTION

An essential component of inertial guidance devices is a gyro, the axis of which defines a fixed direction in space. Considerable progress has been made in the area of rotor suspension by means of gas bearings and electric or magnetic suspension. It is now possible to construct gyros with drifts of less than 0.01 degree/hour, but each unit requires long and careful adjustment. In order to avoid this, attempts have been made to utilize the magnetic properties of atomic nuclei. Various quite different principles are now being studied, one of the most promising apparently being that of the so-called nuclear spin gyro.

1. PRINCIPLE OF THE NUCLEAR SPIN GYRO

1.1 Nuclear resonance

An atomic nucleus may be regarded as a small sphere rotating about its own axis. With respect to this axis it possesses an angular momentum $\vec{I}$ called the spin. This angular momentum creates a

[redacted]

magnetic moment $\vec{M} = \gamma \vec{I}$, γ being the gyromagnetic ratio of the nucleus in question. To simplify the picture, this moment $\vec{M}$ may be regarded as due to the displacement of electric charges at the surface of the sphere.

Let us assume that the moment $\vec{M}$ is placed in a constant magnetic field $\vec{H}_0$. It is then subject to the couple

$$C = \vec{M} \wedge \vec{H} .$$

This couple is equal to the derivative of the angular momentum:

$$\frac{d\vec{I}}{dt} = \vec{M} \wedge \vec{H}$$

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \wedge \vec{H} .$$

The motion of the vector $\vec{M}$ is a gyroscopic motion about the direction of the field H_0 with an angular rate of precession $\omega_0 = \gamma H_0$ (Fig. 1). The corresponding frequency $f_0 = \frac{\omega_0}{2\pi}$ is the Larmor /42 frequency of the nucleus in question. Thus, the spin system has a natural frequency f_0, so that resonance effects can be utilized.

1.2 Application to the gyro [1, 2]

The nuclei employed are the hydrogen nuclei of water (protons) which are placed in the field H_0.

Parallel to this field, we apply an alternating magnetic field $H_1 \cos \omega t$, where ω is close to ω_0. If this system is fixed within a Galilean reference system, no component of the magnetization $\vec{M}$ perpen-

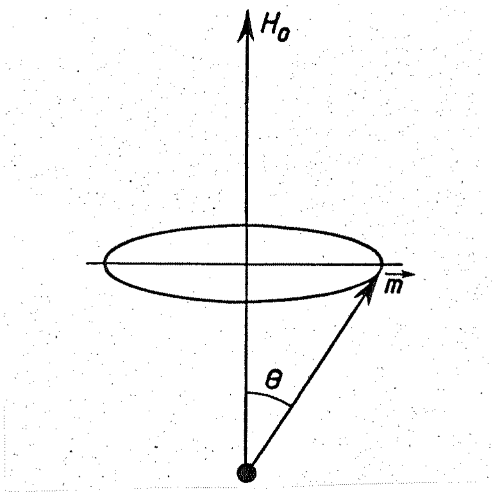

Fig. 1.

dicular to $\vec{H}_0$ is observed. On the other hand, if there is a general rotation about a vector $\vec{\Omega}$ not given by $\vec{H}_0$, it can be shown that the constant magnetic field $\vec{H}_0$ must be replaced in the equations by the sum $\vec{H}_0 + \frac{\Omega}{\gamma}$. Under these conditions, the alternating magnetic field is no longer parallel to the constant magnetic field, and we get a resonance effect expressed in the appearance of components M_x and M_y of the magnetization perpendicular to $\vec{H}_0$ (Figs. 2 and 3).

The principle is identical with that of the following mechanical scheme. Suppose we have a system with a natural frequency f_0 in a plane xoy and a natural frequency, very low compared with f_0, with respect to an axis oz perpendicular to xoy. The system is excited at the frequency f_0 with respect to oz. If this excitation is strictly parallel to oz, no movement is observed in the plane xoy. On the other hand, if

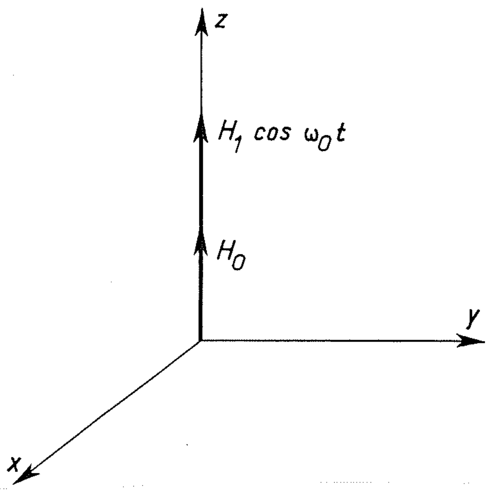

Fig. 2.

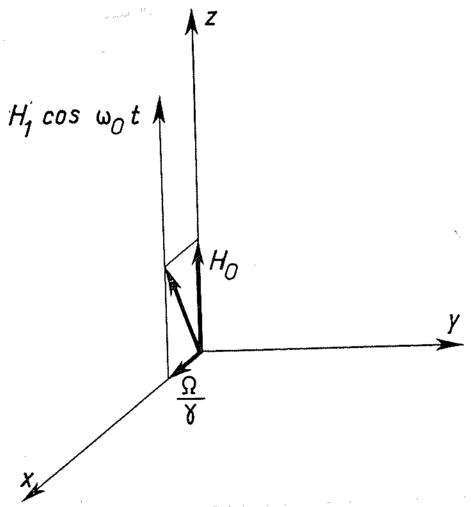

Fig. 3.

it makes a small angle with oz, the component in the plane xoy, being at the natural frequency of the system, will excite resonance, and a considerable movement will be observed in this plane.

In the gyro the method of detecting the existence of the vector $\vec{\Omega}$ is based on a study of the second harmonic of the voltage induced in a coil by the variation in time of the transverse component of $\vec{M}$. The level of this harmonic in the signal varies with the frequency ω, the fields H_1 and H_0, the relaxation time T of the protons, and the rate of rotation Ω. Since the analytic solution of the equations is very difficult, they have been solved on an analog computer, in order to get some idea of the orders of magnitude of the effect.

2. BLOCH EQUATIONS IN NUCLEAR RESONANCE [3, 4]

The magnetic properties of a group of nuclei in an external magnetic field can be represented by a very simple system of equations derived from phenomenological considerations, which we owe to Bloch (1946).

In the case of a constant field H_0 much higher than the alternating field $H_1 \cos \omega t$, the vectorial equation of motion of the magnetization is written:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \wedge \vec{H} - \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{M_z - X_0H_0}{T_1}\vec{k} \qquad (1)$$

where M_x, M_y, M_z are the components of $\vec{M}$ with respect to the trihedral oxyz of unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$, oz being defined by the direction of the field $\vec{H}_0$; T_1 and T_2 are the longitudinal and transverse relaxation times, respectively, and X_0 the static nuclear susceptibility.

This equation ceases to be valid, if the fields H_0 and H_1 are of the same order of magnitude. It can be shown that the magnetization $\vec{M}$ no longer relaxes toward the static value $X_0 \vec{H}_0$, but toward the instantaneous value

$$X_0 [\vec{H}_0 + \vec{H}_1 \cos \omega t].$$

Moreover, under these conditions,

$$T_1 = T_2 = T.$$

The vectorial equation is then:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \wedge \vec{H} - \frac{\vec{M} - X_0 \vec{H}}{T}. \qquad (2)$$

In the case of the nuclear spin gyro, the problem is to know whether the magnetic field $\vec{H}$ forming part of the relaxation term should /43 include the component $\frac{\vec{\Omega}}{\gamma}$ due to the rotation. A study by Heims and Jaymes [6] shows that the physical rotation of a spin system creates a polarization equal to that created by a magnetic field giving a Larmor frequency equal to the frequency of rotation, assuming thermal equilibrium conditions. From this equivalence between the magnetic field and the rotation it is deduced that the component $\frac{\vec{\Omega}}{\gamma}$ should be used in calculating the relaxation term. This also shows that replacing the rotation $\vec{\Omega}$ with a magnetic field $\vec{H}\Omega = \frac{\vec{\Omega}}{\gamma}$, as in the first experiments on the gyro, is correct, since the effects are equivalent for the different terms of equation (2).

3. APPLICATION TO THE NUCLEAR SPIN GYRO

In the nuclear spin gyro, in the presence of a rotation Ω about ox, the components of the magnetic field $\vec{H}$ are:

$$\left\{ \begin{aligned} H_x &= \frac{\Omega}{\gamma} \\ H_y &= 0 \\ H_z &= H_0 + H_1 \cos \omega t. \end{aligned} \right. \tag{3}$$

The vectorial equation (2) is then written:

$$\left\{ \begin{aligned} \frac{dM_x}{dt} &= \gamma M_y (H_0 + H_1 \cos \omega t) - \frac{M_x}{T} + \frac{X_0 v \Omega}{\gamma T} \\ \frac{dM_y}{dt} &= -\gamma M_x (H_0 + H_1 \cos \omega t) - \frac{M_y}{T} + M_z \Omega \\ \frac{dM_y}{dt} &= -M_y \Omega - \frac{M_z}{T} + \frac{X_0 v}{T} (H_0 + H_1 \cos \omega t) \end{aligned} \right. \tag{4}$$

where ν is the volume of the specimen.

The solution of this system of equations would give us the variations of M_x, M_y and M_z as functions of time.

Investigation on an analog computer requires changes in the variables and time calculated to give quantities lying between - 1 and + 1 and sufficiently low frequencies (ω of the order of 1).

3.1 Unit of time

Let us put $\omega = \omega_0 a$, ω_0 being the Larmor frequency linked with the field H_0

$$\omega_0 = - \gamma H_0$$

and substitute for the unit of time $\tau = - \omega_0 t$ $(\omega_0 < 0)$; then

$$\left\{ \begin{aligned} \frac{dM_x}{d\tau} &= M_y (1 + n \cos a\tau) + \frac{M_x}{T\omega_0} - \frac{X_0 \nu \Omega}{\gamma T \omega_0} \\ \frac{dM_y}{d\tau} &= - M_x (1 + n \cos a\tau) + \frac{M_y}{T\omega_0} - M_z \frac{\Omega}{\omega_0} \\ \frac{dM_z}{d\tau} &= M_y \frac{\Omega}{\omega_0} + \frac{M_z}{T\omega_0} + \frac{X_0 \nu H_0}{T\omega_0} (1 + n \cos a\tau) \end{aligned} \right. \qquad (5)$$

where $n = \dfrac{H_1}{H_0}$.

3.2 Change of variables

The existence of a continuous component of the field along the axes ox and oz prompts us to make the change in variables:

$$\left\{ \begin{aligned} M_x &= X_0 \nu \frac{\Omega}{\gamma} (X + 1) \\ M_y &= X_0 \nu \frac{\Omega}{\gamma} Y \\ M_z &= X_0 \nu H_0 (Z + 1). \end{aligned} \right. \qquad (6)$$

Under these conditions, X, Y and Z represent the variable part of M_x, M_y and M_z, with $X_0 \nu \dfrac{\Omega}{\gamma}$ or $X_0 \nu H_0$ taken as the unit, according to the axis.

The equations become:

$$\boxed{\begin{aligned} \frac{dX}{d\tau} &= - Y(1 + n\cos a\tau) - lX \\ \frac{dY}{d\tau} &= - X(1 + n\cos a\tau) - lY + Z - n\cos a\tau \\ \frac{dZ}{d\tau} &= - kY - l(Z - n\cos a\tau) \end{aligned}} \tag{7}$$

where $\quad l = -\dfrac{1}{T\omega_0} \qquad k = +\dfrac{\Omega^2}{\omega_0^2}.$

3.3 Second change of variables

In our experiment the parameters had the following values:

$$T = 333 \ \mu\text{sec} \qquad \omega_0 = 2\pi \times 26.10^3$$

$$\frac{\Omega}{\omega_0} = \frac{1}{28.55}$$

giving

$$l = 0.0184 \qquad k = 0.00123.$$

Since the different terms of the equations must remain less than 1, all the coefficients must be divided by 2.5, since we have the expression n cos a τ with n close to 2. Since the coefficient k then becomes very small, we put

$$Z = lW \tag{8}$$

in order to get a feasible accuracy for this coefficient.

The final system of equations inserted in the machine is then:

$$\begin{aligned} 0{,}4\,\frac{dX}{d\tau} &= 0{,}4\ Y + 0{,}4\,nY\cos a\tau - 0{,}4\,lX \\ 0{,}4\,\frac{dY}{d\tau} &= - 0{,}4\ X - 0{,}4\,nX\cos a\tau - 0{,}4\,lY + \\ &\qquad 0{,}4\ Z + 0{,}4\ n\cos a\tau \\ 0{,}4\,\frac{dW}{d\tau} &= - 0{,}4\,\frac{kY}{l} - 0{,}4\,lW + 0{,}4\ n\cos a\tau \end{aligned} \tag{9}$$

[commas represent decimal points]

the magnetic moments being given by

$$\boxed{\begin{aligned} M_x &= X_0 \rho \frac{\Omega}{\gamma}(X + 1) \\ M_y &= X_0 \rho \frac{\Omega}{\gamma} Y \\ M_z &= X_0 \rho H_0 (lW + 1) \end{aligned}} \qquad (10)$$

4. RESULTS OBTAINED

The results described in this section correspond to the experimental conditions set forth in reference [2]. /44

4.1 Order of magnitude of the variables

In making the above changes of variables, it was assumed that the components M_x and M_y were of the order of $X_0 \nu \frac{\Omega}{\gamma}$. This value is suggested by the orders of magnitude of the variation of the magnetic moment in the absence of a resonance effect. When the machine was started up, the amplification factor of approximately 40 due to resonance was compensated by reducing the scale factor.

4.2 Importance of the second harmonic

The signals X and Y were subjected to harmonic analysis and the various harmonic levels determined.

The following table gives the values obtained:

	X	Y
Fundamental	3	12
2nd harmonic	13.5	7
3rd harmonic	5	7
4th harmonic	2.5	2

In the signal received, which is proportional to the derivatives $\frac{dX}{d\tau}$ or $\frac{dY}{d\tau}$, we get the harmonic levels:

	along Ox	along Oy
Fundamental	5 %	22 %
2nd harmonic	49 %	25 %
3rd harmonic	28 %	38 %
4th harmonic	18 %	15 %

The signal along the axis Ox has a maximum for the second harmonic, this maximum corresponding to the third harmonic along the axis Oy. Figs. 4 through 7 show the shape of the curves obtained for X and Y and their derivatives.

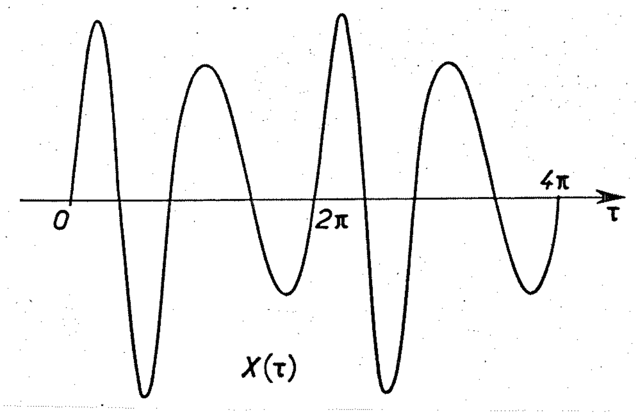

Fig. 4.

4.3 Induced voltages

We have

$$\vec{B} = \mu_0 \vec{H} + 4\pi \vec{J},$$

where

$$\vec{J} = \frac{\vec{M}}{\nu}$$

or

$$\vec{B} = \mu_0 \vec{H} + \frac{4\pi}{\nu} \frac{d\vec{M}}{dt}.$$

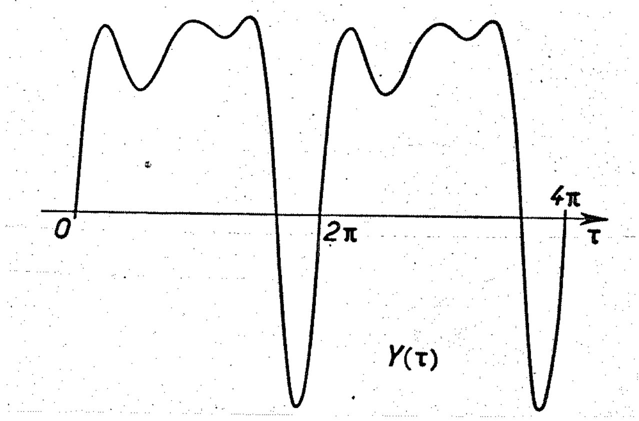

Fig. 5.

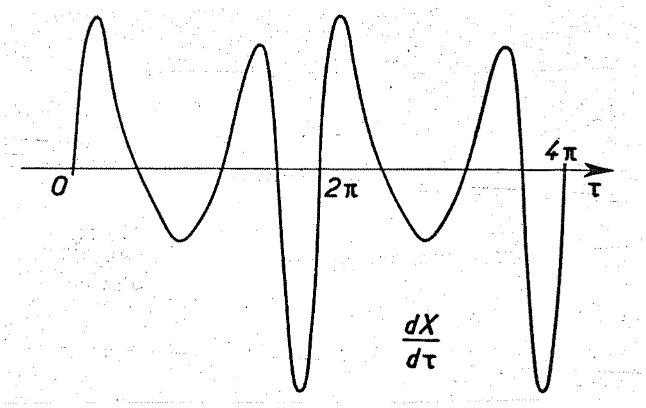

Fig. 6.

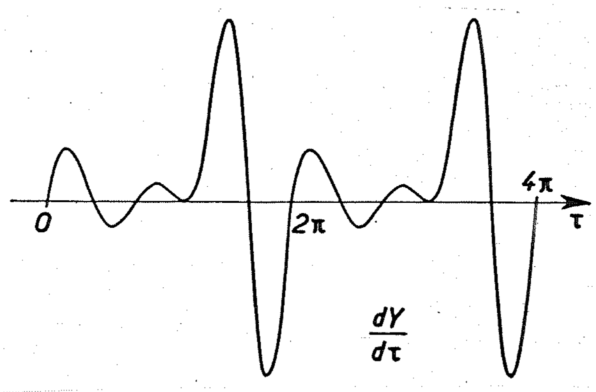

Fig. 7.

On the axes Ox and Oy, /45

$$M_x = X_0 \rho \frac{\Omega}{\gamma} (X + 1)$$
$$M_y = X_0 \rho \frac{\Omega}{\gamma} Y$$

whence

$$\frac{dM_x}{dt} = X_0 \rho \frac{\Omega}{\gamma} \omega_0 \frac{dX}{d\tau}$$
$$\frac{dM_y}{dt} = X_0 \rho \frac{\Omega}{\gamma} \omega_0 \frac{dY}{d\tau}.$$

If S is the surface area of the pickup coil, the induced voltage will be

$$V_x = 4\pi X_0 S \omega_0 \frac{\Omega}{\gamma} \frac{dX}{d\tau}$$
$$V_y = 4\pi X_0 S \omega_0 \frac{\Omega}{\gamma} \frac{dY}{d\tau}.$$

Measured on the second harmonic of the signal induced in the coil, the effective voltages will be:

$$\boxed{\begin{aligned}(V_x)_2 &= 4\pi X_0 S \omega_0 \frac{\Omega}{\gamma}\left(\frac{dX}{d\tau}\right)_2 \\ (V_y)_2 &= 4\pi X_0 S \omega_0 \frac{\Omega}{\gamma}\left(\frac{dY}{d\tau}\right)_2\end{aligned}}$$

Numerical example.

Taking V in volts and the other quantities in the system cgs em, we have:

$$X_0 = 3 \cdot 10^{-10}$$

$$S = 10 \text{ cm}^2 \text{ (for one turn)}$$

$$\frac{\Omega}{\gamma} = 0.2 \text{ gauss}$$

$$\omega_0 = 1.6 \cdot 10^5$$

$$(V)_2 = 0.13 \cdot 10^{-10} \left(\frac{d}{d\tau}\right)_2 .$$

Substituting in this expression the values obtained for the harmonics, we get:

$$(V_x)_2 = 0.35 \cdot 10^{-9} \text{ volt}$$

$$(V_y)_2 = 0.18 \cdot 10^{-9} \text{ volt.}$$

In order to obtain signals of the order of some ten microvolts, it would thus be necessary to provide coils with about 1,000 turns, assuming a quality factor of 50 (tuned coil).

5. EFFECT OF PARAMETERS

The equations obtained depend on four parameters. The analog computer makes it possible to vary them very simply by regulating the various potentiometers, so that the optimum values can be determined.

5.1 Variation of n

By definition $n = \dfrac{\text{crest value of alternating field}}{\text{continuous field}}$. The field ratio was modified so as to get $1 < n < 2.5$. Qualitatively, we observed:

- A rapid increase in the peak amplitude of the signals $\dfrac{dX}{d\tau}$ and $\dfrac{dY}{d\tau}$ for n varying between 1 and 2, then a slower increase for n varying between 2 and 2.5.

- A regular deformation of the shape of the same signals with variation in n. For n close to 2 there are two practically equal maxima on the $\dfrac{dX}{d\tau}$ curve, which may correspond to a minimum of the first harmonic for the signal $X(\tau)$.

- It was directly verified that the component in $\cos 2\tau$ of the signal $\dfrac{dX}{d\tau}$ passes through a maximum at $n = 2$.

5.2 Modification of the initial conditions

The values of the variables in the steady regime were obtained from the initial conditions:

$$X = 0 \qquad Y = 0 \qquad W = 0$$

or

$$M_x = X_0 \nu \frac{\Omega}{\gamma} \qquad M_y = 0 \qquad M_z = X_0 H_0.$$

The initial conditions were modified with respect to the three variables, and in every case the results relating to the steady regime remained identical. The time required to establish the steady regime is of the order of 10 periods, which is in agreement with the ratio $\frac{\omega_0}{2\pi T}$ given by the theory.

5.3 Variation of Ω

The factor Ω modifies the coupling between Y and Z, since it appears in the parameter $k = \frac{\Omega^2}{\omega_0^2}$.

The following table gives the variations in the peak-to-peak amplitude of the signal $\frac{dX}{d\tau}$ as a function of Ω, Ω_0 corresponding to the experimental conditions described in reference [2].

Ω	0	$\Omega_0/2$	Ω_0
crest value of $(\frac{dX}{d\tau})$	150	130	120
$\frac{\Omega}{\Omega_0} \frac{dX}{d\tau}$	0	65	120

Thus, the curve of the crest voltage, proportional to $\Omega \frac{dX}{d\tau}$, as a function of Ω is practically a straight line in the region investigated. Accordingly, the crest voltage is proportional to Ω, and the result also appears to be valid for the second harmonic, since the shape of the curve $\frac{dX}{d\tau}$ varies little as a function of Ω (Fig. 8).

5.4 Width of resonance line (Figs. 9 and 10)

The width of the resonance line was measured for two values of the relaxation time T. /46

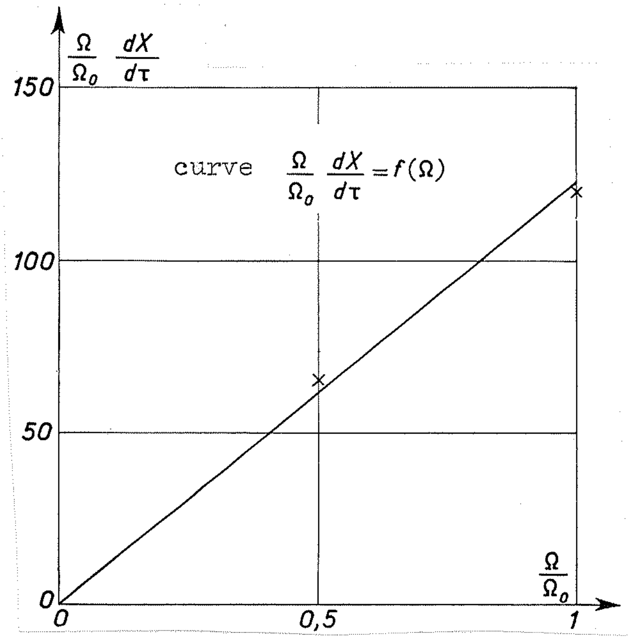

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Fig. 8.

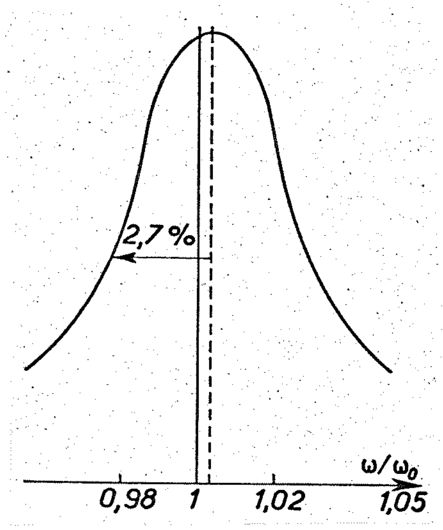

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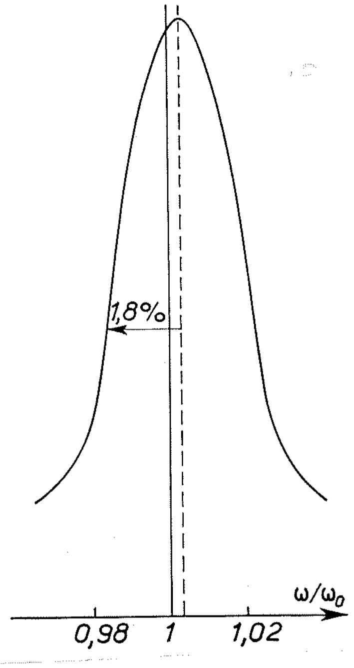
Fig. 9. Resonance line, T = 330 μsec.



[commas represent decimal points]

Fig. 10. Resonance line, T = 660 μsec.

T = 330 μsec: assuming a line of the Lorentz type, we get a relative half-width of 2.7%. The theoretical value corresponding to the relaxation time is $\frac{1}{T\omega_0}$ or 1.8%.

T = 660 μsec: with the same assumptions, we get a relative half-width of 1.8% instead of 0.90%.

For both cases the central frequency corresponds to: $a = \frac{\omega}{\omega_0} = 1.003$. Bearing in mind the existence of the fields $\frac{\Omega_0}{\gamma}$ and H_1, the corresponding theoretical frequency is: $a \sim 1.002$.

5.5 Remarks

The variation of ω_0 is identical with the simultaneous variation of the parameters l, n and k. Combining the above results, we can thus

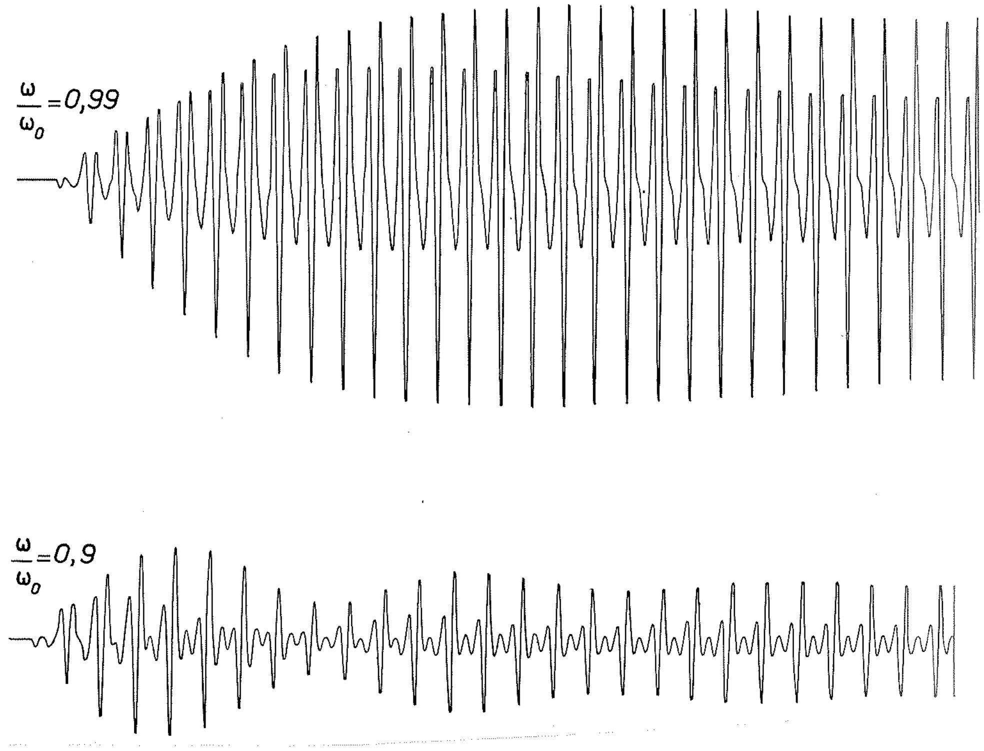


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Fig. 11. Examples of transient regime for $\frac{dx}{d\tau}$.

get the shape of the signals for a different Larmor frequency.

The curves obtained for X, Y and W have characteristics in the transient regime that depend heavily on the various parameters (presence of oscillations, relative amplitudes of maxima and minima, phase of the observed signals relative to the excitation signal).

All these properties are unexplained and almost impossible to allow for in the equations (Fig. 11).

6. CONCLUSIONS

6.1 The orders of magnitude obtained make it possible to establish a certain number of requirements relating to the experimental device.

a) The relative width of the resonance line is of the order of one percent. Thus, it is necessary to ensure the stability of the directional field H_0, and hence of the excitation frequency ω_0, with an accuracy of 0.1%. It is also possible to envisage a sweep of a few percent of the field H_0 in order to stay in resonance.

b) The ration $n = H_1/H_0$ has little influence on the results, if n remains close to 2 with a possible error of 5%.

On the other hand, the residual level of the second harmonic in the exciting signal should be maintained at as low a value as possible. If the device for controlling the orthogonality of the emitting and receiving coils is of the order of a minute of arc, the leakage signal from the emitting coil to the receiving coil will be 100 times higher than the resonance signal. This leads to a reduction in the distortion level to a value of less than 1% [6]. /47

c) The parallelism of the fields H_1 and H_0 must be perfect, if a signal is not to be observed in the absence of rotation. The accuracy of the first measurements did not make it necessary to investigate this question very thoroughly, the production of the two fields by means of the same coils being deemed sufficient.

d) If it is intended to use the gyro as a zero detector for a servo-platform, the response time of the gyro must be included in the servo response time. This response time is of the order of magnitude of the relaxation time and should therefore remain within reasonable limits.

Furthermore, the perturbation signal $\frac{\Omega}{\gamma}$ modifies the resonant frequency of the system. In order to get a sufficiently high signal amplitude, this modified frequency must still be close to the maximum of the resonance line. Since the width of this line is proportional to 1/T, T must be small.

e) Taking as the noise factor of the pickup coil the thermal noise for a resistance of the order of 1 kΩ and assuming a transistorized tuned preamplifier with a noise factor of 3 db, we may anticipate the detection of signals of less than 1 μv. The results mentioned in reference [2] envisage a signal of 6 μv. We hope to get a signal of the same order of magnitude without too much trouble.

6.2 The field $\frac{\vec{\Omega}}{\gamma}$ employed corresponds to a rotation of 900 revolutions per second. It appears that one can gain:

- a factor of 10 with respect to the detection of the signal;

- a factor of 4 with respect to the product of the overvoltage factor and the fill factor;

- a certain factor with respect to the electronics by reducing the pass band.

Thus, there is some hope of obtaining rotations of the order of 10 revolutions per second, which would be close to the practical limits of real rotation for the present size of the apparatus.

The results obtained above are valid for experiments conducted at the ambient temperature. A further step would be transition to the low temperatures necessitated by the requirements of magnetic shielding. Under these conditions, it is possible to count on an improvement in sensitivity due to the following factors:

a) An increase in X_0 which is proportional to $\frac{N}{t^\circ K}$ (Curie's law). With helium 3, we would have $X_0 \# 10^{-9}$, or a gain of 10.

b) The thermal noise power is divided by 100. By using superconducting shielding and taking precautions to eliminate microphonic noise, it is possible to take advantage of this reduction.

c) The overvoltage factors of coils in non-superconducting material do not increase if the temperature is reduced (experiments conducted in Professor Grivet's laboratory at Orsay).

Using superconducting material makes it possible to obtain values /48
of more than 5,000 without special difficulty.

d) The effect of the relaxation time T of the nuclei has not been completely elucidated. It is to be expected, however, that if T increases, it will be possible to obtain greater signal amplitudes. In fact, saturation only appears at $T \sim \frac{1}{\Omega}$. For normal rates of rotation, one could take a T of the order of 0.01 sec.

The total anticipated gain on transition from 300°K to 3°K is thus of the order of 10^6.

We must express our thanks to Professor Benoit for his assistance in establishing the equations and analyzing the results, and Mr. Bouttes

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